Elementary Mathematics Notes

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1 Numbers & Their Operations

Types of numbers

Integers ($\mathbb{Z}$): $..., -3, -2, -1, 0, 1, 2, 3, 4, ...$

Prime: integers that are divisible by 1 and itself only, smallest prime number is 2

Rational numbers ($\mathbb{Q}$) $\frac{\text{integer}}{\text{integer}}$: $\frac{4}{7}, -3\frac{1}{8}, 0.3, 2.\dot{6}\dot{5}, 92, \sqrt{16}$

Irrational numbers: $\pi, \sqrt{2}, e$

Real numbers ($\mathbb{R}$): all numbers

Standard form

$A \times 10^n$, where n is an integer, and $1 \leq A < 10$

SI prefix

Prefix	10^n
pico	10^{-12}
nano	10^{-9}
micro	10^{-6}
milli	10^{-3}
kilo	10^{3}
mega	10^{6}
giga	10^{9}
tera	10^{12}

Indices

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^m = a^m b^m$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6. $a^{-n} = \frac{1}{a^n}$
7. $a^0 = 1$
8. $a^{\frac{1}{n}} = \sqrt[n]{a}$
9. $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

2 Ratio & Proportion

Map scale

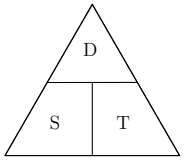
Length scale $= 1 : r$

Area scale $= 1 : r^2$

3 Percentage

$$\text{Percentage increase / decrease} = \frac{\text{increase / decrease}}{\text{original}} \times 100\%$$

4 Rate & Speed



$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

5 Algebraic Expressions & Formulae

n^{th} term

$a + (n-1)d$

Special algebraic identities

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

6 Equations

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

7 Set Language & Notation

$\in$: is an element of

n(A): number of elements in set A

A': complement of set A

$\varnothing$: empty set

ξ: universal set

$\cup$: union

$\cap$: intercept

$\subset$: subset

8 Problems In Real-World Contexts

Simple interest

$$I = \frac{PRT}{100}$$

Compound interest

$$A = P\left(1 + \frac{R}{100}\right)^n$$

9 Angles, Triangles & Polygons

Types of polygons

No. of sides	Polygons
3	triangle
4	quadrilateral
5	**pent**agon
6	**hex**agon
7	**hept**agon
8	**oct**agon
9	**non**agon
10	**dec**agon

Sum of interior & exterior angles

Sum of interior angles $= (n-2) \times 180°$

Sum of exterior angles $= 360°$

10 Congruence & Similarity

Congruent & Similar Triangles

Congruent triangles	Similar triangles
SSS, SAS, AAS, RHS	SSS, SAS, AAA

Ratio of area & volume

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

11 Pythagoras' Theorem & Trigonometry

Pythagoras' theorem

$a^2 + b^2 = c^2$

Trigonometric ratios

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

TOA CAH SOH is applicable for only right-angled triangles

Obtuse angles

$\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$

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Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

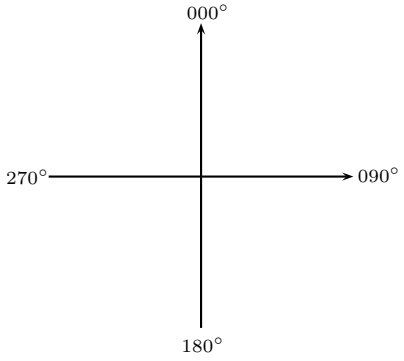
Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

Area of triangle

Area of triangle $= \frac{1}{2}ab\sin C$

Bearings



A bearing is a **3-digit positive number** with units of degree to show direction **clockwise** from the north direction.

12 Mensuration

Conversion

1 m = 100 cm
1 m^2 = 10,000 cm^2
1 m^3 = 1,000,000 cm^3

Radian & Degree

$180° = \pi$ rad

Arc length & sector area

Degree

$s = \frac{\theta}{360°} \times 2\pi r$, where θ is in degrees
$A = \frac{\theta}{360°} \times \pi r^2$, where θ is in degrees

Radian

$s = r\theta$, where θ is in radians
$A = \frac{1}{2}r^2\theta$, where θ is in radians

13 Coordinate Geometry

Cartesian coordinate

(x, y)

Gradient

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Equation

$y - y_1 = m(x - x_1)$
$y = mx + c$
*Vertical line: $x = a$
*Horizontal line: $y = b$

Length

Length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

14 Vectors In 2 Dimensions

Representation

Vectors can be represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$, $\mathbf{a}$ or $\underset{\sim}{a}$.

Magnitude

$|\overrightarrow{AB}|$ or $|\mathbf{a}| = \sqrt{x^2 + y^2}$

15 Data Analysis

Mode

Mode is the **most frequently occurring** number. A set of data can have **more than one** mode.

Mean

$$\text{mean} = \frac{\text{sum of all numbers}}{\text{number of numbers}}$$

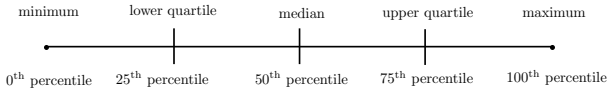
Median

Median is the **centre** number when the numbers are arranged from **smallest to largest**.

Range

Range = maximum − minimum

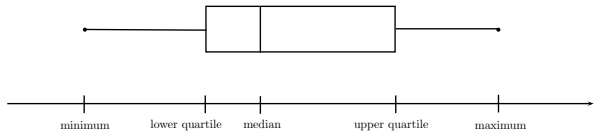
Quartiles & percentiles



Interquartile range

Interquartile range = upper quartile − lower quartile

Box-and-whisker plot



Mean & standard deviation

Ungrouped

Mean, $\bar{x} = \frac{\sum x}{N}$
Standard deviation, $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$

Grouped

Mean, $\overline{x} = \frac{\sum fx}{\sum f}$
Standard deviation, $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$